High-frequency blockade in a tight-binding one-dimensional lattice with single vibrating atomic state

M.V. Entin* and M.M. Mahmoodian†
Institute of Semiconductor Physics, Siberian Division,
Russian Academy of Sciences,
Novosibirsk, 630090 Russia

One-dimensional tight-binding lattice, single site of which possesses harmonically vibrating level is studied. The states of non-interacting electrons incident with fixed energy from infinity are considered. It is shown that at definite conditions the site reflects electrons *absolutely and elastically* (high-frequency blockade states). The problem is treated both numerically and (in the case of narrow band) analytically. The results are compared with the free-electron 1D problem with vibrating δ-functional potential. Together with the blockade states the local and reflectionless states are examined. Possible realization of the system as a lattice of quantum dots is discussed.

PACS numbers: 73.50.Pz, 73.23.-b, 85.35.Be

The transport through systems with local vibrating potential attracted attention in connection to the quantum pumps [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]. One interesting feature of these systems is the presence of states with no transmission (high-frequency blockade states) [12, 13, 14, 15, 16, 17, 18, 19, 20]. The blockade states are unusual and impossible in conservative systems. They give a lot of new possibilities of control of electron transport in nanostructures. In particular, the absolute reflection by a single vibrating well leads [19, 20] in double-well structure to appearance of local states situated on the background of continuum. These states produce very narrow controlled resonances in the system transparency.

Here we concentrate on the problem of electron states in the tight-binding model with the vibrating level of a single site. This problem permits both numerical and, in the case of narrow permitted band, analytical consideration. We shall study the problems of continuous (scattering) and discrete eigenstates. The model permits coexistence of such discrete states as blockade, reflectionless and local states which will be the topic of our special attention. From this standpoint, the tight-binding model is much reacher than the considered earlier models. In addition, we shall discuss the possibility of practical realization of systems with local vibrating potential.

We study a 1D tight-binding lattice expressed by a system of equations

$$i\dot{a}_m - \delta_{m,0}(u + v\cos(\omega t))a_0 + \frac{\Delta}{2}(a_{m+1} + a_{m-1}) = 0. \quad (1)$$

The Eq. (1) corresponds to a lattice with the overlapping amplitudes between sites $\Delta/2$, the energy is referred to the levels of sites with $n \neq 0$. We use the quantities $u,\ v,\ \Delta,\ E$ measured in units of frequency ω; $\hbar = 1$.

In absence of the vibrating site ($u = v = 0$) the eigenstates of the system $a_m = e^{ipm - iE(p)t}$ with quasimomentum p has the energy $E(p) = -\Delta\cos p$. The propagating electron can possess energy within the permitted band $-\Delta < E < \Delta$.

If $v = 0,\ u \neq 0$ the site $n = 0$ represents an "impurity" which produces scattering of propagating electrons and the impurity state above or below the permitted band. In presence of vibrations ($v \neq 0$) the energy does not conserve changing by some quanta ω.

We consider two kinds of states, namely, the scattering problem with given energy of an incident electron and local quasienergy (Floquet) states; the latter are permitted in the tight-binding model, unlike the free-electron model with a vibrating delta-potential.

The general solution of the scattering problem reads

$$a_{m\leq 0} = \sum_n \left(\delta_{n,0}e^{ip_0 m} + r_n e^{-ip_n m}\right) e^{-i(E+n)t},$$
$$a_{m\geq 0} = \sum_n t_n e^{ip_n m} e^{-i(E+n)t}. \quad (2)$$

Here $r_n = t_n - \delta_{n,0}$, p_n is the solution of the equation $E + n = -\Delta\cos p_n$ corresponding to the positive velocity of the states inside the permitted band, in accordance with causality. Some of states get to the forbidden band; for them the positive imaginary value of p_n should be chosen to guarantee the decay of states apart from the zero site.

The transition amplitudes t_n satisfy an equation

$$t_n(\Lambda(E+n) - u) - \frac{v}{2}(t_{n+1} + t_{n-1}) = \Lambda(E)\delta_{n,0}. \quad (3)$$

Here $\Lambda(E) = i\sqrt{\Delta^2 - (E + i\cdot 0)^2}$ (the term $i\cdot 0$ reflects the causality). In general, the system (3) can be solved numerically.

The system under consideration can be treated as a quantum wire constructed from a long chain of quantum dots the ends of which are connected to electron seas (source and drain). Suppose, that the source and the drain of the device have equilibrium electron gas at zero temperature. The chemical potentials of seas are different due to the applied dc voltage. The solutions of the scattering problem determine the system conductance:

$$G = G_0 \int dE \sum_n |t_n|^2 \left(-\frac{\partial f(E)}{\partial E}\right). \quad (4)$$

Here $f(E)$ is the Fermi distribution function; $G_0 = 2e^2/h$ is the conductance quantum.

The presence of mentioned blockade states reflects in the vanishing of the conductance at definite conditions

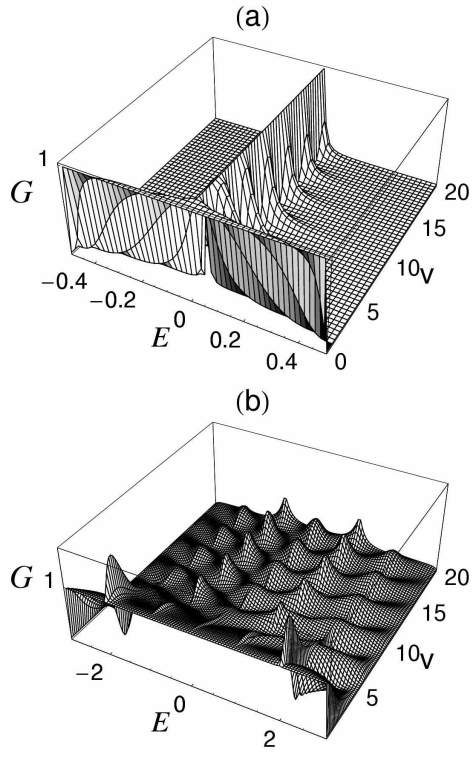

FIG. 1: Conductance G (in units of G_0) versus the Fermi energy and parameter v at $u = 0$, $\Delta = 0.5$ (a) and $\Delta = 2.5$ (b). The Fig. (a), where $\Delta < 1$, distinctly shows the reflectionless state $E = 0$, $G = 1$. For $\Delta > 1$ (Fig. (b)) the reflectionless state disappear.

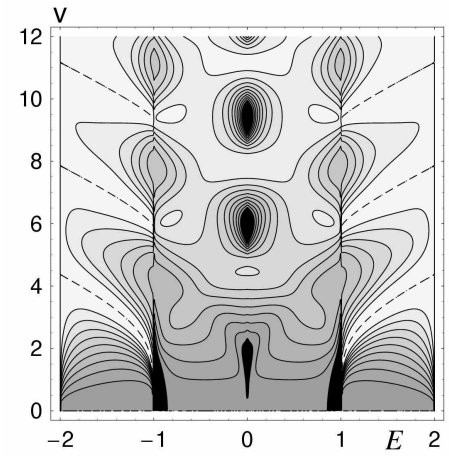

FIG. 2: Map of conductance levels (in units of G_0) as a function of the Fermi energy and the parameter v at $u = 0$, $\Delta = 2$. The levels run equidistantly from $< 10^{-6}$ (white) to ≥ 1 (black). The blockade states are depicted by dashed curves.

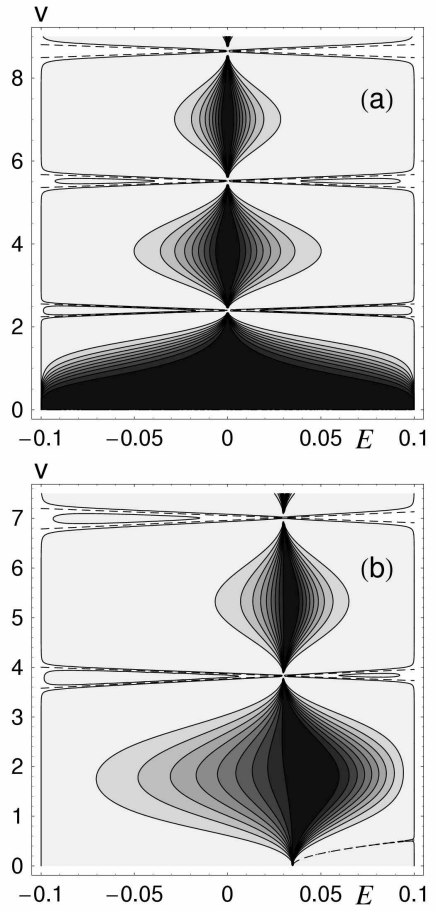

FIG. 3: The same as in Fig. 2 at $\Delta = 0.1$ and $u = 0$ (a), $u = 0.13$ (b). In the last case the picture loses symmetry $E \leftrightarrow -E$. Blockade states cross all permitted band.

Numerical results

The calculated conductance at zero temperature is depicted in the Figs. 1-4 for different values of Δ. The Fermi energy, denoted below as E, runs from the bottom $-\Delta$ to the top Δ of the permitted band.

The conductance oscillates with v, that reflects the interference nature of the process. At $u = 0$ the pictures are symmetric with respect to the band center (Figs. 1, 2). This electron-hole symmetry can be easily deduced from the Eq. (3). For large $\Delta > 1$ (Figs. 1b, 2) the picture exhibits singularities, connected with the thresholds $\pm\Delta$, namely $\pm(\Delta - n)$. The singularities $n = 0, 1$ are most pronounced. The other singularities (easing with n) arise as photon repetitions of these singularities. For a wide permitted band $\Delta \gg 1$ the behavior of the conductance near the band boundaries is similar to the model of free electrons interacting with the vibrating δ-potential [19, 20]. This is not the case for a narrow permitted band.

There are two other important features of the conductance dependence. One is the vanishing of the conductance at specific energies $E(u, v)$ (blockade states). It means that at corresponding energy all channels of transmission become closed, $t_n = 0$ for $|E + n| < \Delta$.

These high-frequency blockade states can be found directly. Note that if for some $n > 0$ $t_n = t_{n+1} = 0$, then all $t_k = 0$ for $k > n + 1$. Hence, the condition $t_0 = t_1 = 0$ automatically solves all equations (3) with $n > 0$. The equations for $n < 0$ compose a homogeneous system, while the equation for $n = 0$ determines the value for t_{-1}. This condition can be fulfilled if $-\Delta + 1 > E > -\Delta$. If the energy satisfies the condition $\Delta - 1 < E < \Delta$, above mentioned is valid with the change of sign in front n. The homogeneous system has solutions for some eigenvalues of parameters. Practically, they can be found as

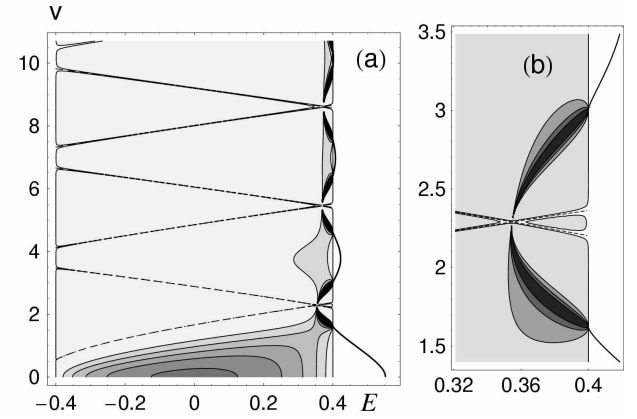

FIG. 4: (a): The same as in Fig. 2 at $\Delta = 0.4$, $u = 0.38$. In this case the crossing point of the blockade and reflectionless states approach the permitted band boundary $E = 0.4$. The local states (solid lines) are situated to the right of this boundary. (b): Magnified neighborhood of the first crossing point of blockade states and reflectionless states illustrates how the local states convert to the reflectionless states (inside black regions).

eigenvalues for $1/v$, if one rewrites the system as

$$\frac{1}{v}t_n - \frac{1}{2(\Lambda(E+n)-u)}(t_{n+1}+t_{n-1}) = 0, \qquad n < 0. \quad (5)$$

The Figs. 2, 3a show the dependence of blockade states on parameters for $u = 0$. If $\Delta > 1$ (Fig. 2) the domains of existence of the blockade states tight to the left and right band boundaries do not overlap, if $\Delta < 1$ (Fig. 3a) the blockade states cross, and if $\Delta < 1/2$ they overlap all permitted band.

Another feature is the existence of a state with absolute transmission when the conductance exactly equals to 1. For $u = 0$ this reflectionless state corresponds to the center of permitted band $E = 0$. It exists if $\Delta < 1$. It is interesting that there are essentially singular points where the blockade and reflectionless states coexist.

For $u \neq 0$ the relief of the conductance becomes asymmetric and, in particular, the line of the reflectionless state moves from the permitted band center $E = 0$ and curved.

For finite u besides the considered states the local states appear in the forbidden band. These states are constructed from the waves decaying apart from the site $n = 0$. They obey the Eq. (2) with no incident wave for energy which lies in the forbidden band (all p_n are chosen with positive imaginary part). These states originate from the local impurity state at $v = 0$ when the transitions into propagating modes caused by alternating field are forbidden by the energy conservation law. Accordingly, their quasienergy plus (or minus) any integer number should not get into the permitted band. The local states appear below or above the forbidden band in dependence on the sign of u.

Similar to resonance tunneling in conservative systems, the local states transform to the reflectionless states when they go through the permitted band boundaries $E = \pm\Delta$ (Fig. 4).

Analytical results

Let us consider the limit $\Delta \ll 1$. For states with energy $E \gg \Delta$ in the zero approximation one can replace $\Lambda(E)$ by E, and if $E \ll 1$ rewrite the system (3) as

$$(H-\lambda)_{n,n'}\, t_{n'} = \Lambda(E)\delta_{n,0}, \quad (6)$$

where $\lambda = u - E$ and $H = H^{(0)} + V$,

$$H^{(0)}_{n,n'} = n\delta_{n,n'} - \frac{v}{2}(\delta_{n,n'-1} + \delta_{n,n'+1}). \quad (7)$$

The perturbation V reads as

$$V_{n,n'} = (\Lambda(E+n) - E - n)\delta_{n,n'}. \quad (8)$$

In the limit of small Δ all elements of $V_{n,n'}$ are negligible except for $n = 0$, and

$$V_{n,n'} = (\Lambda(E) - E)\delta_{n,0}\delta_{n',0}. \quad (9)$$

The Eq. (6) with local perturbation Eq. (9) can be solved using the Green function. The eigenvectors ψ_k of the Hamiltonian $H^{(0)}$ are given by the integer Bessel functions $\psi_k = J_{n-k}(v)$ whose eigenvalues are k. The Green function $\hat{G}^{(0)} = 1/\hat{H}^{(0)}$ is

$$G^{(0)}_{n,n'} = \sum_k \frac{J_{n-k}(v)J_{n'-k}(v)}{k-\lambda}.$$

Using the Green function, we find

$$t_n = \frac{\Lambda(E)\sum_k \frac{J_{n-k}(v)J_{-k}(v)}{k-\lambda}}{1 + [\Lambda(E) - E]\sum_k \frac{J^2_k(v)}{k-\lambda}}. \quad (10)$$

In the considered case only the channel $n = 0$ belongs to the propagating states. Hence the zeros of t_0 determine the blockade states, while the poles do the localized states:

$$\sum_k \frac{J^2_k(v)}{k-\lambda} = 0 \qquad \text{for blockade states,} \quad (11)$$

$$1 + [\Lambda(E) - E]\sum_k \frac{J^2_k(v)}{k-\lambda} = 0 \qquad \text{for local states.} \quad (12)$$

Blockade states. If $u \ll 1$ then the terms $k = 0$ with the denominator $\lambda = u - E$ majorizes the sums, and Eq. (11) gives the energies of the blockade states. For $v \ll 1$ one can keep two terms of the series in Eq. (11) with $k = 0$ and $k = N$. As a result we obtain

$$E_0 = \tilde{u} + \frac{N}{(N!)^2}\left(\frac{v}{2}\right)^{2N} \quad (13)$$

The blockade states also exist near the points $v = v_{0,m}$, where $v_{n,m}$ are the zeros of the Bessel functions $J_n(v)$. If u is close to some integer N: $u = N + \tilde{u}$, $\tilde{u} \ll 1$ we obtain

$$E_m = \tilde{u} \pm \frac{J_N(v)}{\left[\sum_{k\neq N} J^2_k(v_{N,m})/(k-N)^2\right]^{\frac{1}{2}}}. \quad (14)$$

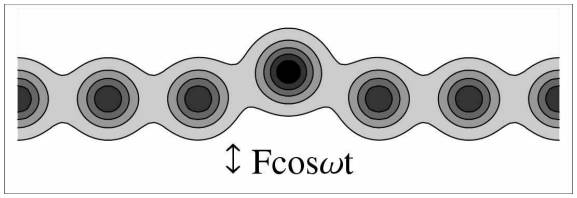

FIG. 5: Curved 1D quantum dot lattice as an example of considered model. The potential $U(x,y) = -w\sum_{n\neq 0}\cosh^{-2}\sqrt{(x-na)^2+y^2}/l - (w+u)\cosh^{-2}\sqrt{x^2+(y-b)^2}/l$ is supposed. The Figure represents potential relief for $a=1$, $b=0.3$, $l=0.3$, $w=1$, $u=0.2$.

Local states. The local states satisfy to an equation

$$u = N + E\left[1 - \left(1 - \sqrt{1-\Delta^2/E^2}\right) J_N^2(v)\right]. \quad (15)$$

If the alternating signal $v=0$, the local states convert to the static impurity states $E = u\sqrt{1+\Delta^2/u^2}$. For weak, but finite v, the level move to the boundaries of the permitted band:

$$E = u\left[\sqrt{1+\Delta^2/u^2} - \frac{v^2}{2}\left(1 + \frac{1}{\sqrt{1+\Delta^2/u^2}}\right)\right]. \quad (16)$$

This result can be obtained from the perturbation series on the alternating signal.

The generalization of Eq. (16) for the case of large v reads

$$E = \tilde{u}\left[1 + \frac{J_N^2(v)}{2J_N^2(v)-1}\left(1 + \sqrt{1+[2J_N^2(v)-1]\frac{\Delta^2}{\tilde{u}^2}}\right)\right] \quad (17)$$

In general, the subsequent local states appear (disappear) when the energy achieve the permitted band boundaries $|E| = \Delta$. This gives the threshold equations

$$u = N \pm \Delta[1 - J_N^2(v)]. \quad (18)$$

The numerical results are in accord with the obtained formulae.

How to realize local high-frequency perturbation?

The most interesting events considered here happen when the frequency is comparable with the band width and the Fermi energy. The application of so high frequency is difficult by using of ordinary electrodes. More natural way is utilization of a freely propagating electromagnetic wave. (Neglecting the corrections caused by low-dimensional system itself). The wavelength of electromagnetic wave for typical frequency $\omega \gtrsim E$ exceeds the electron wavelength, that mean non-locality of electromagnetic perturbation. Nevertheless, the ways for locality exist. For example, it can be done using the curved system, by analogy with the way to produce effectively-nonuniform magnetic field.

Let us consider the 1D periodic chain of quantum dots with a single quantum dot number 0 shifted in y direction in the distance b (see Fig. 5). Being placed into alternating electric field $F\cos(\omega t)$ in y direction this system simulates the considered tight-binding model with parameter $v = Fb$. The action of the field on other quantum dots can be neglected if it is out of resonance with distance between their levels.

The planar lattice of quantum dots looks quite common, that does the considered abstract model realistic. Thus, the local probing of the system by the optical-frequency electric field becomes possible. The careful examination of this model goes beyond the topic of the present paper.

In conclusions, we have demonstrated that the local vibrating potential in 1D system can act as an ideal mirror, despite the openness of the system. These blockade states co-exist with the states of ideal transparency and local states.

The work was supported by grant of RFBR No 08-02-00506, the grant of the President of the Russian Federation No MK-271.2008.2 and the grant of the Russian Science Support Foundation.

* Electronic address: entin@isp.nsc.ru
† Electronic address: mahmood@isp.nsc.ru

[1] D. J. Thouless, Phys. Rev. B **27**, 6083 (1983).
[2] P. W. Brouwer, Phys. Rev. B **58**, R10135 (1998).
[3] M. Switkes, C.M. Marcus, K. Campman, and A.C. Gossard, Science **283** 1905 (1999).
[4] S. W. Kim, Int.J.Mod.Phys. B **18**, 3071 (2004).
[5] M. Moskalets and M. Büttiker, Phys. Rev. B **66**, 205320 (2002).
[6] M. Moskalets and M. Büttiker, Phys. Rev. B **69** 205316 (2004).
[7] Shi-Liang Zhu and Z. D. Wang, Phys. Rev. B **65**, 155313 (2002).
[8] S. W. Kim, Phys. Rev. B **66**, 235304 (2002).
[9] L. S. Braginskii, M. M. Makhmudian and M. V. Entin, Zh. Eksp. Teor. Fiz. **127**, 1046 (2005) [JETP **100**, 920 (2005)].
[10] M. M. Mahmoodian, L. S. Braginsky, M. V. Entin, Phys. Rev. B **74**, 125317 (2006).
[11] M. M. Mahmoodian, M. V. Entin, Europhys. Lett. **77**, 67002 (2007).
[12] P. F. Bagwell and R. K. Lake, Phys. Rev. B **46**, 15329 (1992).
[13] M. Wagner, Phys. Rev. B **49**, 16544 (1994).
[14] M. Wagner, Phys. Rev. A **51**, 798 (1995).
[15] W. Li and L. E. Reichl, Phys. Rev. B **60**, 15732 (1999).
[16] D. F. Martinez and L. E. Reichl, Phys. Rev. B **64**, 245315 (2001).
[17] O. A. Tkachenko, V. A. Tkachenko, and D. G. Baksheev Phys. Rev. B **53**, 4672 (1996).
[18] O. A. Tkachenko, V. A. Tkachenko, and D. G. Baksheev Phys. Rev. B **54**, 13452 (1996).
[19] M. M. Makhmudian, M. V. Entin, and L. S. Braginskii, Zh. Eksp. Teor. Fiz. **132**, 561 (2007) [JETP **105**, 495 (2007)].
[20] M. M. Mahmoodian, M. V. Entin and L. S. Braginsky, Physica E **40**, 1205 (2008).